

## Qn 1 → Introduction to Combinatorics (61)

S4CS

- Qs.1) What is Sudoku Puzzle? write its benefits?
- The Modern version of Sudoku was invented in 1979 by Howard Garns in USA & called Number Place. The word Sudoku is an abbreviation of a phrase which means "the digits must occur only once in a row". The aim of Sudoku puzzle is to fill in the grid so that each row, each column & each box contains all the numbers from 1 to 9.

Benefits :-

- ① Solving Sudoku is fun, exciting & mental challenge. It is addictive in good sense.
- ② Cognitive Assessment : It develops the students' skill in identifying no in specific columns & retaining those nos wherever necessary.
- ③ Elimination :- By eliminating which nos are already given in the particular square, the student can employ the process of 'eliminating' those given no to arrive at a particular no.
- ④ Logical thinking :-
- ⑤ Analysis :-
- ⑥ Number sense :-

Q1) In a state of Maharashtra, license number consists of 2 digit followed by a space followed by capital letters. The first digit cannot be 0. How many license nos are possible?

$\Rightarrow$  Soln:  $X = \{1, 2, 3, \dots, 9\}$   
 $S = \{1, 2, 3, \dots, 9\}$   
 $X = \text{Space} = 1$   
 $2 = \text{Alphabet}$

$\{X-0^3 X \times X^4 \times 2^2\}$   
 number

$$\begin{aligned}\therefore \text{No of diff license number} &= \{X-0^3 X \times X^4 \times 2^2\} \\ &= 9 \times 10 \times 1 \times 26 \times 26\end{aligned}$$

Q2) Suppose that a website allows user to set password with condition, the first character must be a lowercase letter in the English alphabet, second and third must be upper or lower case letters or decimal digit (0 to 9). Fourth place must be @. Fifth & sixth are lowercase English letter, \*, %, and #. # and seventh place must be a digit. How many different passwords can a user set?

$\Rightarrow$  Soln: To prepare a string of length 7

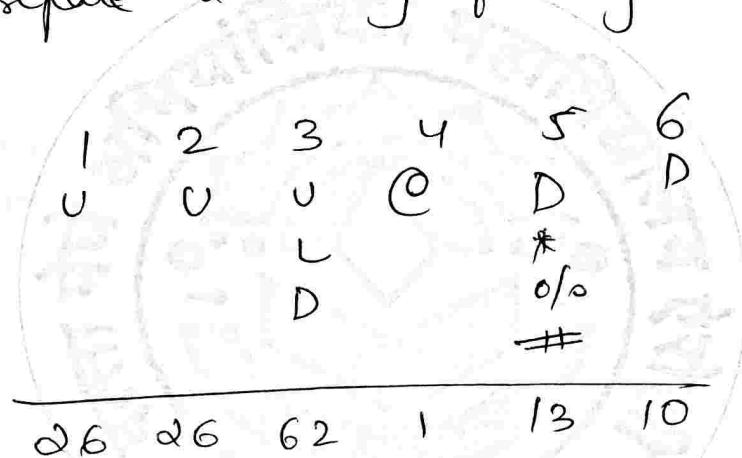
1	2	3	4	5	6	7
v	L	L	@	L	L	D
L	v	v	*	*	*	
D	D		%	%		
			#	#		

$$\begin{array}{r} 26 \\ + 26 \\ + 26 \\ + 10 \\ \hline 62 \end{array}$$

$$2) \quad 26 \times 62 \times 62 \times 1 \times 29 \times 2^9 \times 10$$

$$\Rightarrow \underline{\underline{840529040}}$$

(Q3) Suppose that a bank allow user to set user ID for website with condition , the first or second character must be a upper case letter in the English alphabet , third letter must be a vowel or upper case alphabet or decimal digit (0 to 9) fourth place must be @ . Fifth & sixth must be a digit or \* , % , # & both must be a digit ? Now many different user ID can user set  
 $\Rightarrow$  to prepare a string of length 6



$$\therefore \text{Diff user ID} = 26 \times 26 \times 62 \times 1 \times 13 \times 10 \\ = 5448560$$

## Ques on Combinations

- (5) A Maharashtra restaurant has 11 items in the vegetable category of its menu. A one vegetable plate consists of different vegetable. How many different ways can a customer order?
- (6) There are 10 questions in question paper. In how many diff ways can a student attempt 7 questions from the paper?
- (7) A box contains 6 white balls & 4 black balls. In how many ways can one select 3 white & 2 black balls from the box?
- (8) There are 5 professors & 5 students. A committee of 6 persons is to be formed taking atleast 2 persons from each group. Find the no of such possible committees.
- (9) The staff of department consist of a manager, an officer and 10 clerks. A committee of 4 is to be selected from the department. Find the no of ways in which this can be done so as to always include (i) the manager.  
(ii) manager but not officer  
(iii) neither manager nor officer.
- (10) There are 8 professors & 12 students out of whom a committee of 2 professors & 3 students is to be formed. Find the no of ways in which the committee can be formed such that  
(i) a particular professor is included  
(ii) a particular ~~professor~~ student is excluded.

(11) A group consist of 10 person out of whom 6 are men & 4 are women. In how many ways can a committee of 5 person be selected so that the men are in majority?

\* (12) Find the no of ways in which a selection of 4 letters can be made from the letters of word MATHEMATICS?

(13) A farmer buys 3 cows, 2 goats & 4 hen from a man who has 4 cows, 3 goats & 8 hens. How many choices does the farmer have?

(14) How many numbers are there between 100 & 1000 in which all the digits are distinct?

(15) How many diff no can be formed from the digits 0, 2, 3, 4, 5, 6 lying between 100 & 1000 in which no digit being repeated? How many of them are not divisible by 5?

(16) How many four digits number can be formed from the digits 1, 2, 3, 4, 5 with repetition possible which are divisible by 5?

(17) Three sets A, B, C contain 4, 5, 6 taken respectively. In how many ways can a person select 7 tokens by choosing atleast 2 tokens from each set?

(18) In Supreme court bench consist of 5 judges. Find no of ways in which the bench can give a majority decision?

\* Binomial Theorem:

- ① Find coefficient of  $x^5y^7$  in  $(3x-2y)^{12}$
- ② Find coefficient of  $x^9y^3$  in  $(x-7y)^{12}$
- ③ Find coefficient of  $y^2x^7$  in  $(4x+7y)^9$
- ④ Find coefficient of  $xy^8$  in  $(x+y)^{10}$
- ⑤ Find coefficient of  $x^5y^8$  in  $(2x-3y)^{13}$
- ⑥ Find third term in the expansion of  $(2x-3y)^5$
- ⑦ Multinomial coefficient what is the coefficient of  $x^{66}y^{64}z^{28}$  in  $((3x)^3+y-z^2)^{100}$ ?
- ⑧ What is the coefficient of  $x^7y^2z^{66}$  in  $(5x^2+y+z^5)^{50}$ ?
- ⑨

\* First principle of mathematical induction:-

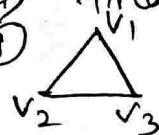
- ① Prove that sum of first  $n$  natural numbers is  $\frac{n(n+1)}{2}$
- ② Prove by MI,  $1+2+3+\dots+(3n-1) = \frac{n(3n+1)}{2}$
- ③ Prove that sum of squares of first  $n$  natural numbers is  $\frac{n(n+1)(2n+1)}{6}$
- ④ Prove that  $1+3^2+3^3+\dots+3^n = \frac{3}{2}(3^n-1)$

- (5) Prove by MI  $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$
- (6) Prove by MI  $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} < \frac{n}{n+1}$

### \* Second principle of Mathematical Induction

- (1) If  $u_1=3, u_2=5, u_n=3u_{n-1}-2u_{n-2} \forall n \geq 3$   
 Then using induction, show that  $u_n=2^n+1 \forall n \geq 3$
- (2) Let  $a_n$  be the recursive relation defined by  
 $a_n=da_{n-1}+a_{n-2} \forall n \geq 2$  with initial conditions  
 $a_0=1, a_1=2$ . Prove that  $a_n \leq \left(\frac{5}{2}\right)^n$

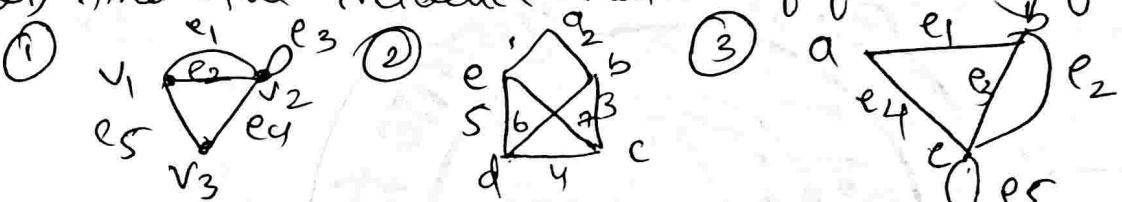
Give an example of a graph with 5 vertices & 5 edges  
and also represent in oriented pair

- (Q1) Find the adjacent matrix of following graph:
- ① 
  - ② 
  - ③ 
  - ④ 

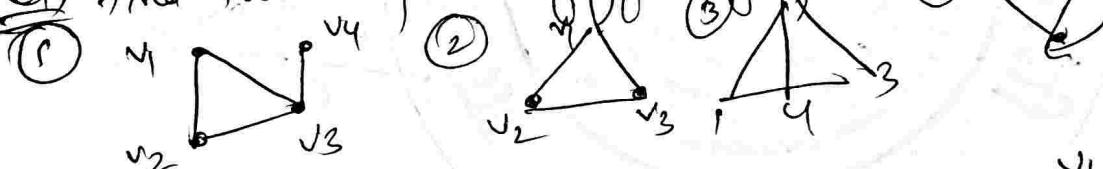
- (Q2) Given the adjacency matrix, draw the graph

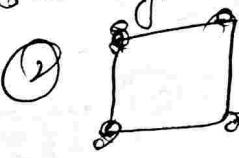
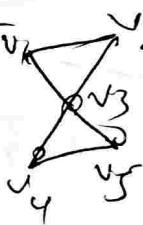
$$\text{A}(G) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- (Q3) Find the incidence matrix of following graph:



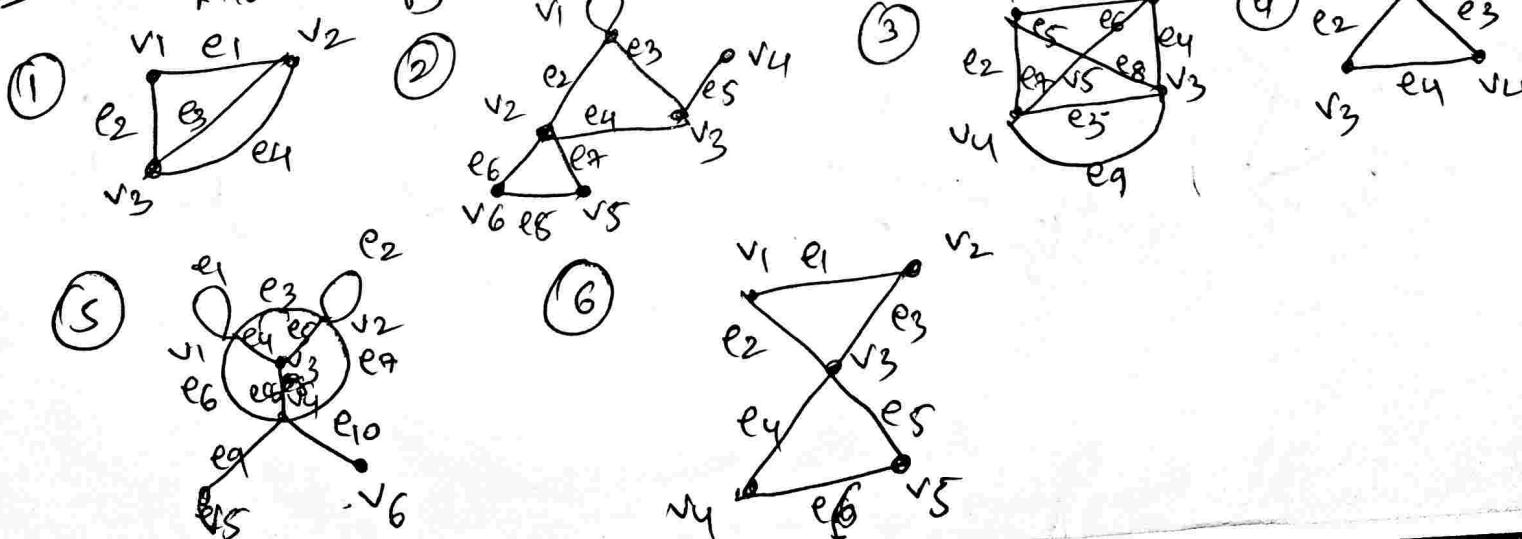
- (Q4) Find the degree of following graph:



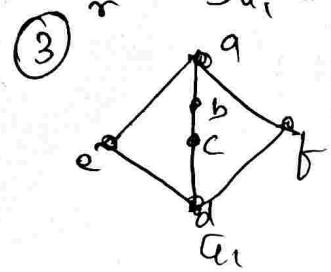
- (Q5) Verify Hand Shaking Lemma
- ① 
  - ② 
  - ③ 
  - ④ 

### Assignment - 1

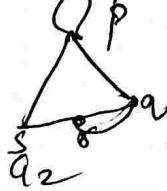
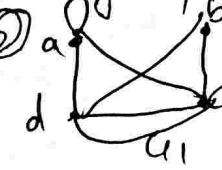
- (Q1) Find the adjacent & incident matrix of following graphs.  
Also verify Hand Shaking Lemma



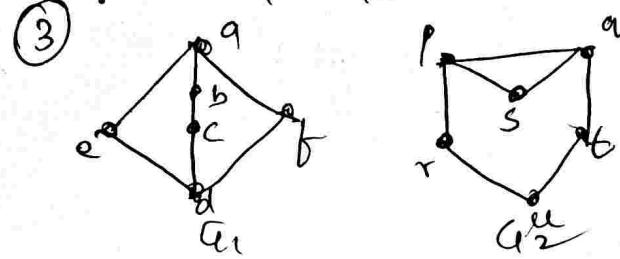
\* Isomorphism  
Check whether the following graphs are isomorphic to each other



$G_2$ :  $\square$  with diagonal  $p-q$

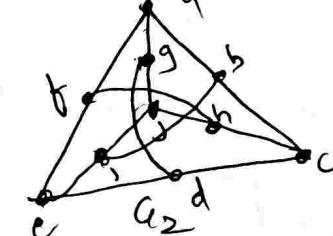
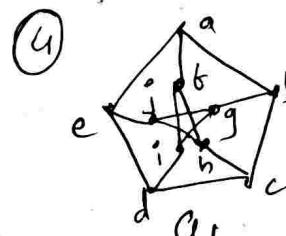
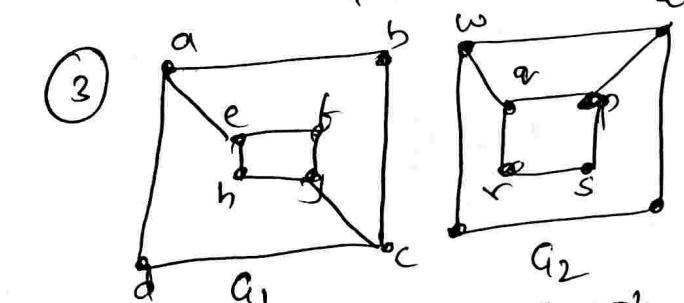
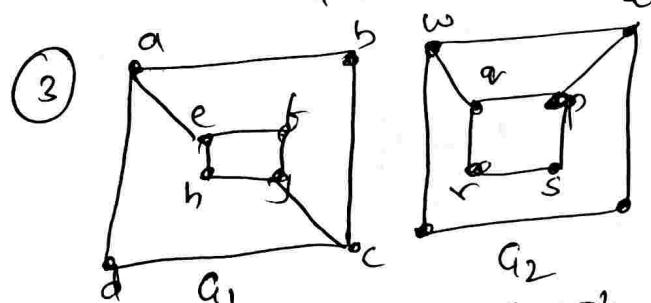
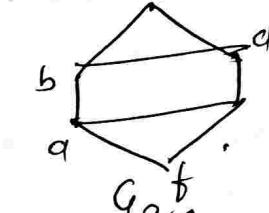
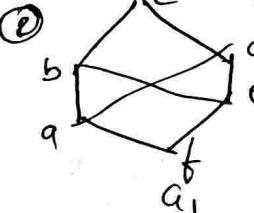
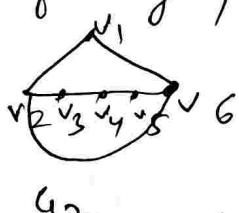
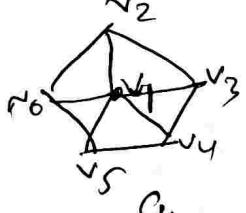


$G_3$ :  $\square$  with diagonal  $a-b$



$G_2$ :  $\square$  with diagonal  $s-t$

Assignment - 2  
Check whether the following graphs are isomorphic to each other



\* complete graph, Regular graph  $\rightarrow 10/07/20$

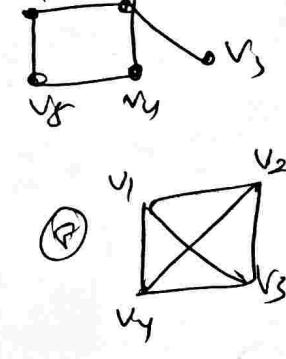
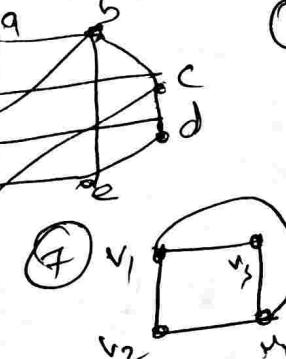
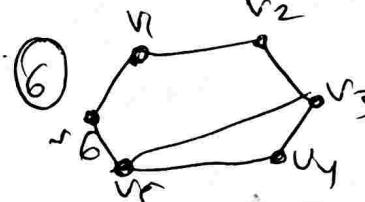
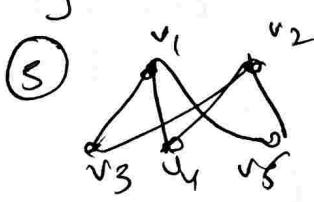
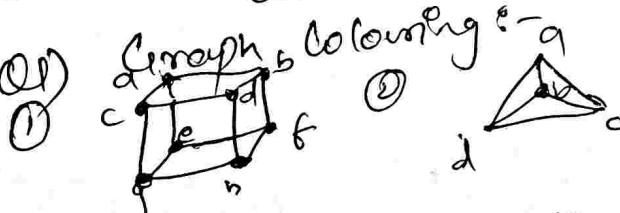
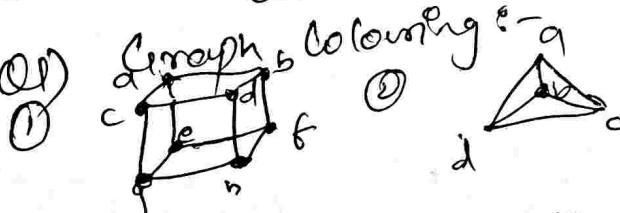
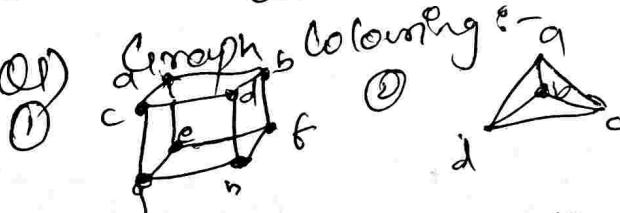
\* Wall, Total, Path

① Give example of graph  
Eulerian & Hamiltonian  
Hamiltonian but not Eulerian

② Eulerian & not Hamiltonian

③ Neither Eulerian nor Hamiltonian

④ Graph Colouring :-  
G1: 3 colours  
G2: 2 colours  
G3: 4 colours

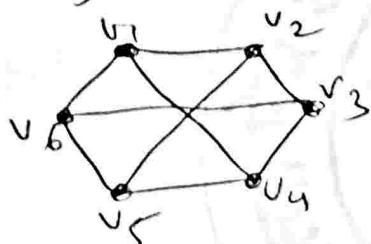
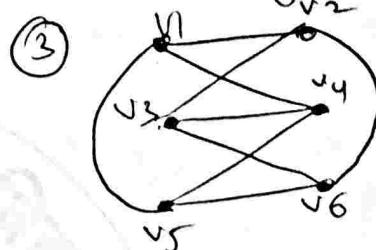
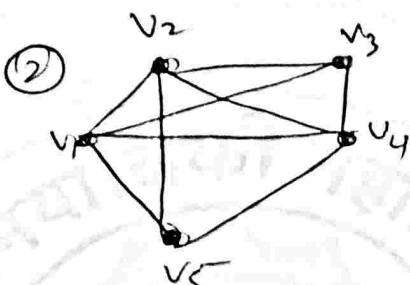
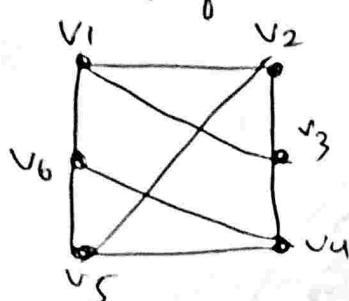


represent  $K_4$  in planar representation

- ①  $K_5$  is not planar
- ②  $K_{3,3}$  is not planar.

### Assignment - 3

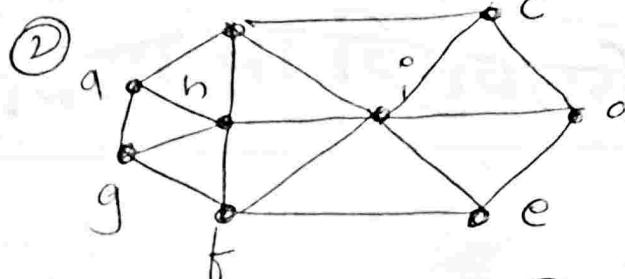
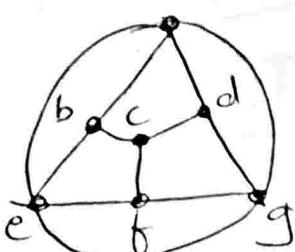
Draw the following planar graphs of no. 2  
Also find the chromatic no. of foll graphs



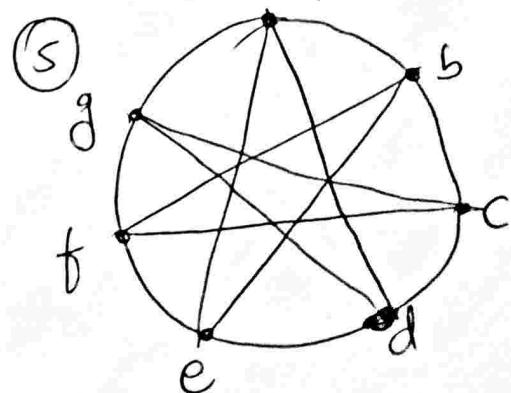
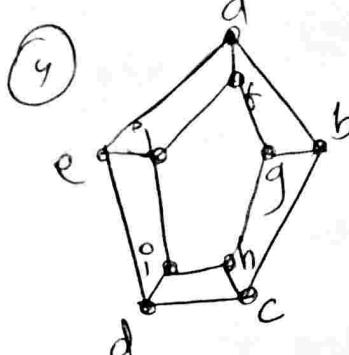
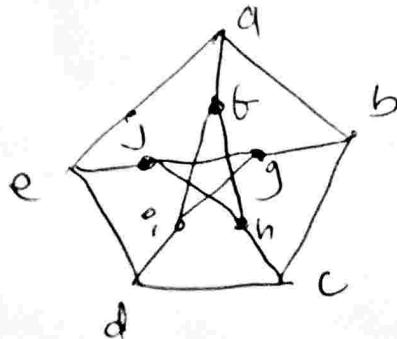
### Assignment - 4

chromatic no. of foll graphs:

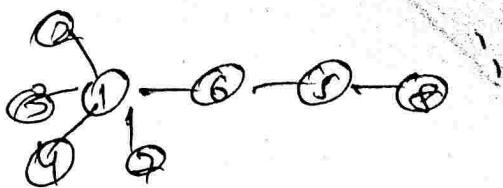
①



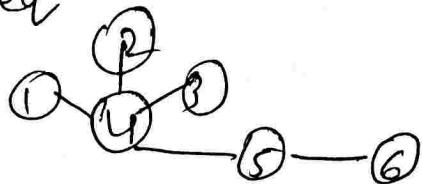
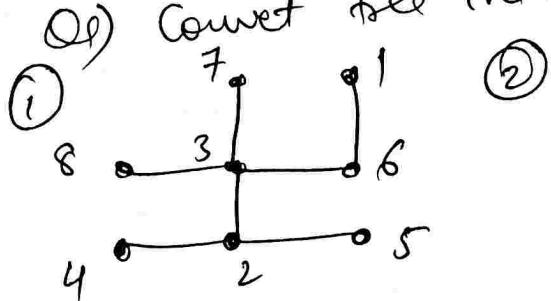
③



Q) Convert seq into pre  
① (4, 1, 3, 4)      ② (1, 1, 1, 1, 6, 5)

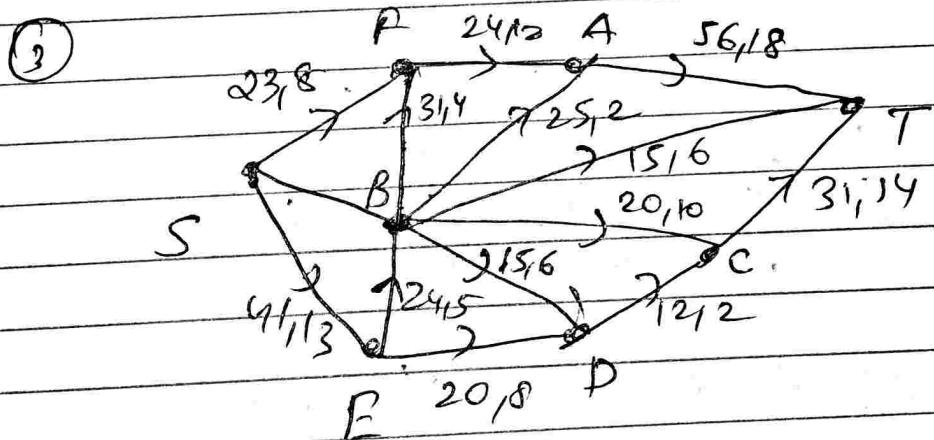
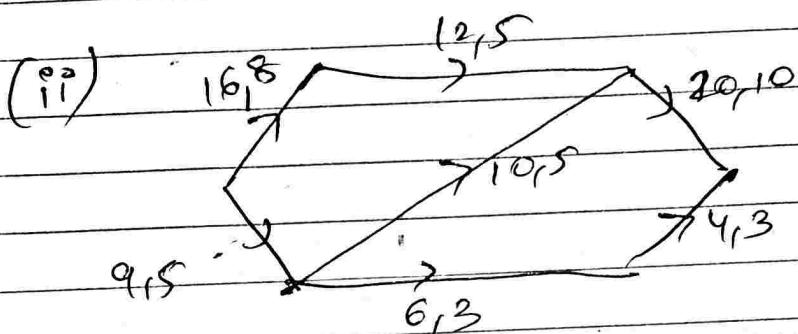
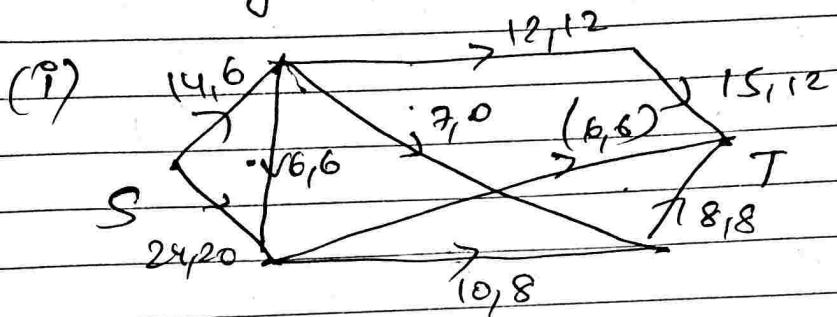


Q) Convert pre into seq



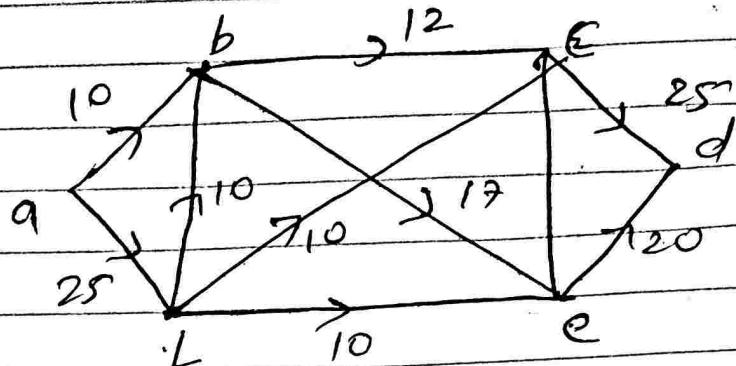
## Unit 3

- (Q1) What are the basic notation & terminology in network flow?
- (Q2) Explain flow & cuts? Give the relation between flows & cuts?
- (Q3) Explain augmenting path?
- (Q4) Define capacity of the cut?
- (Q5) Find capacity of different cuts possible in diagram?



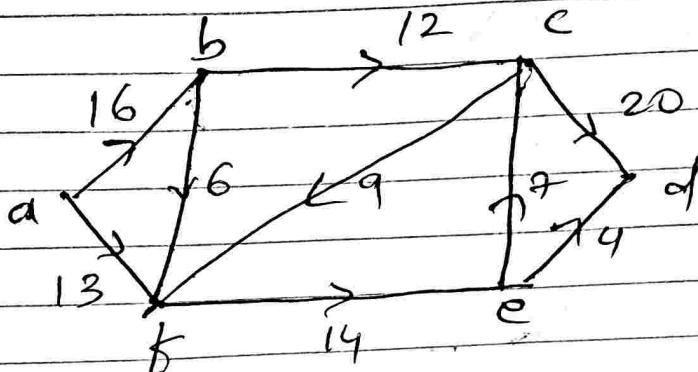
(Q5) Ford Fulkerson Cabiling Algorithm :-

①

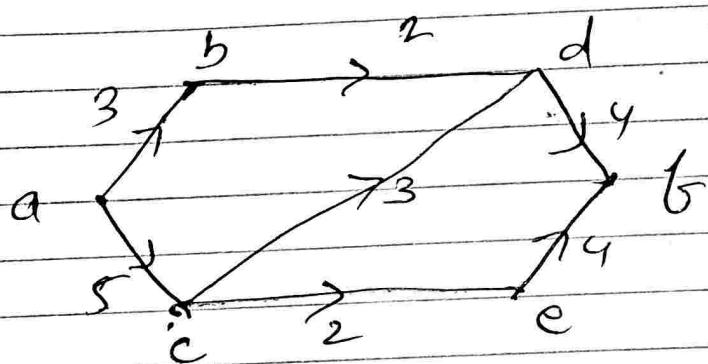


Find value of max flow?

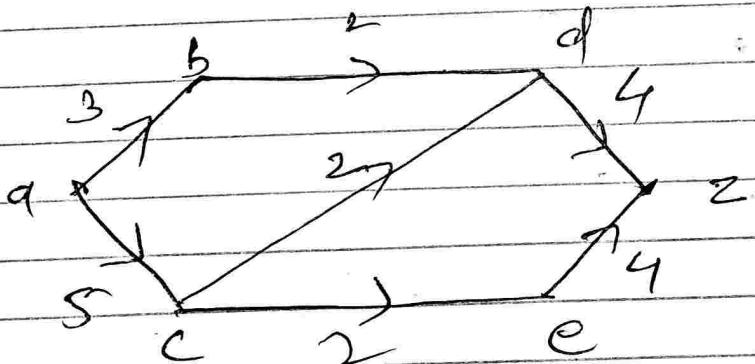
②



③



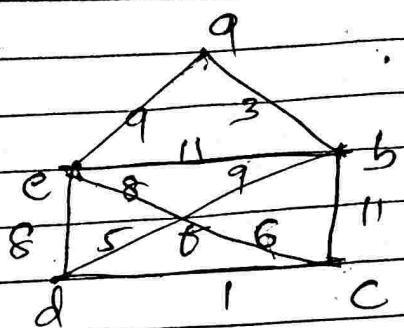
④



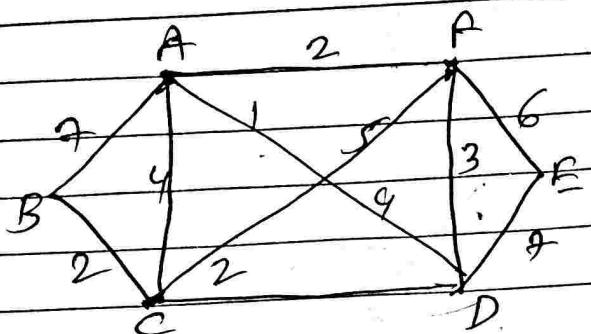
Q7) Kruskal Algorithmus:

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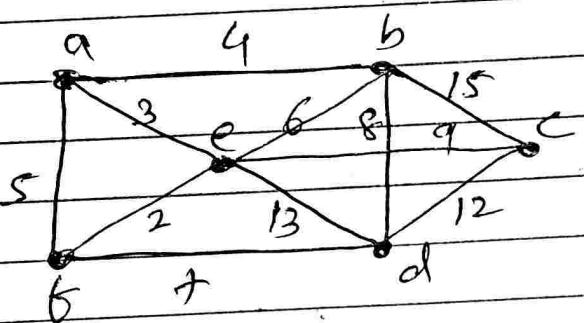
(1)



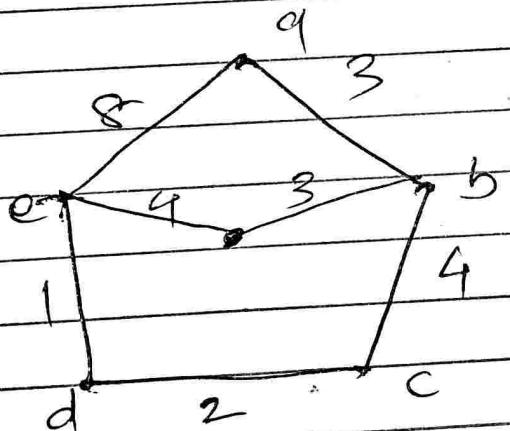
(2)



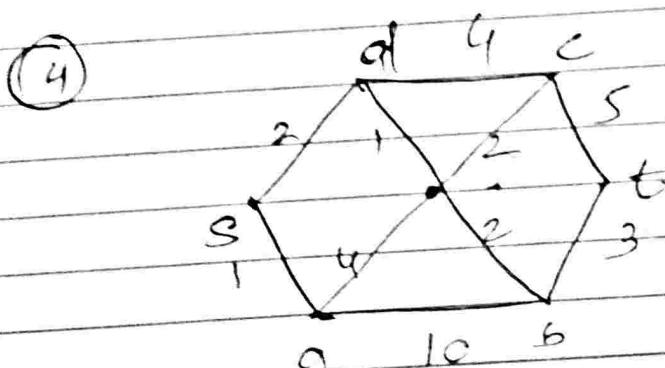
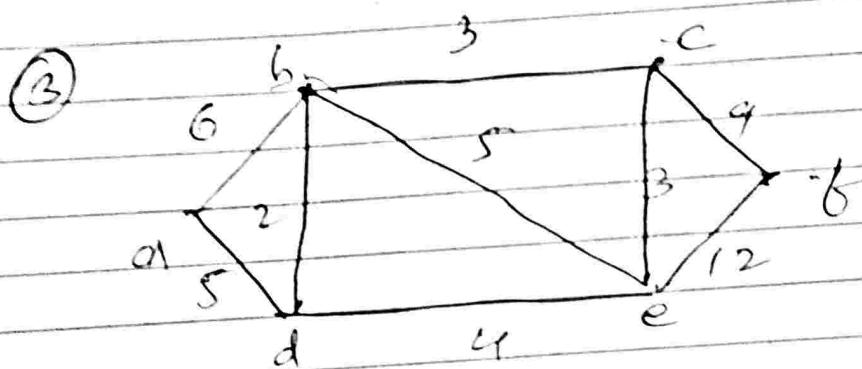
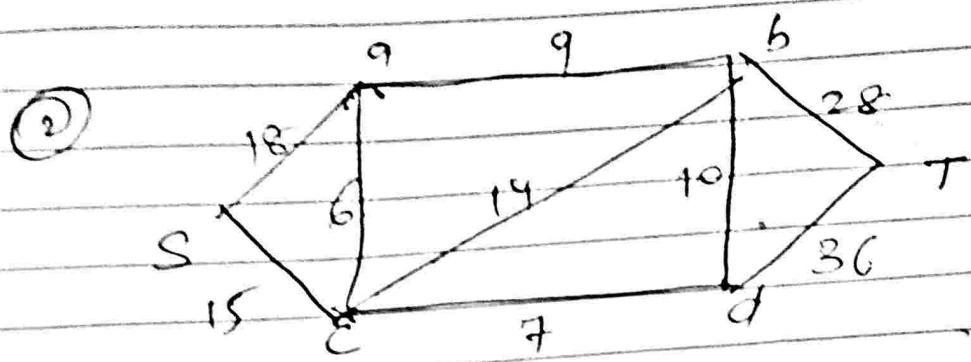
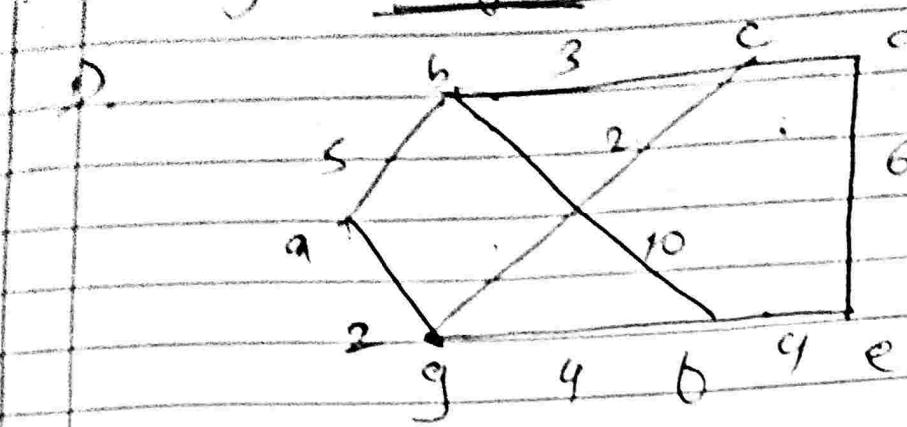
(3)



(4)

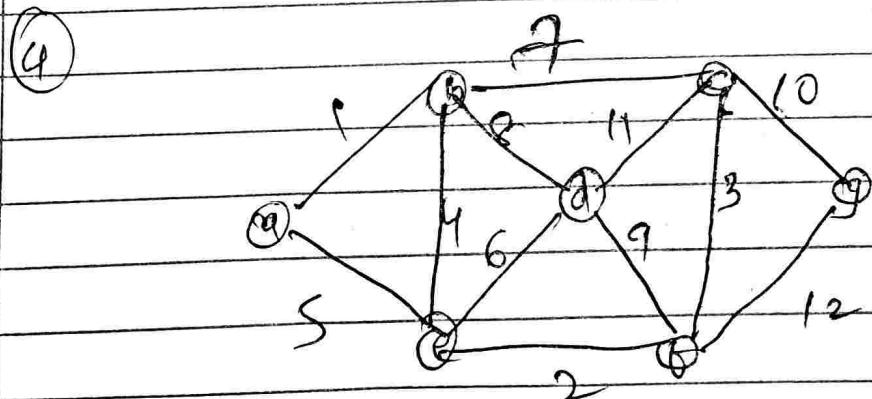
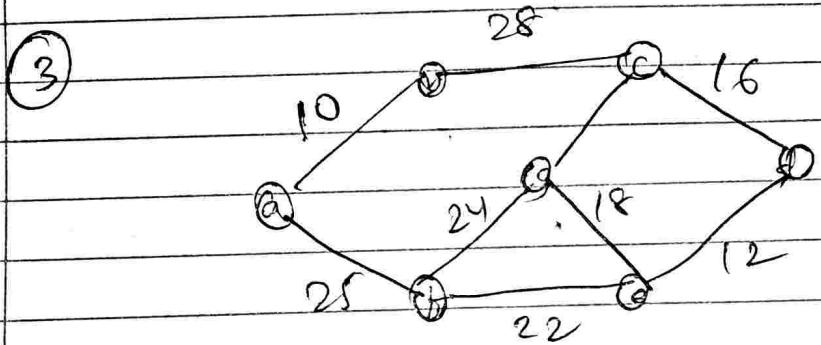
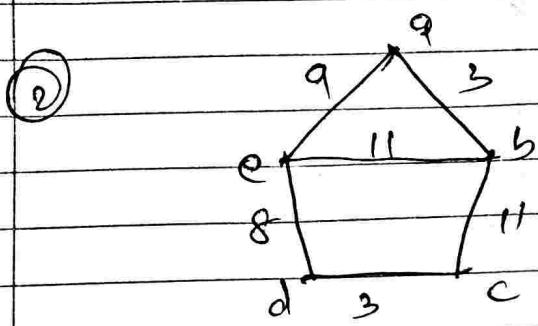
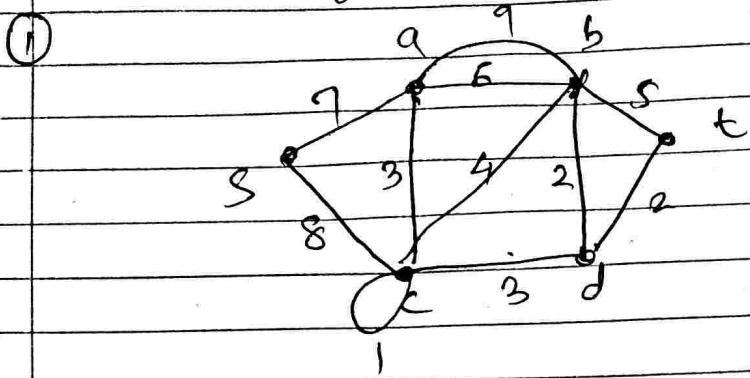


(S3) Diktat Algorithmen



Q9) Brute Algorithmen

10



Q(10) Explain Poly's Enumeration Theorem ?

Q(11) Define Chain Partitioning

Q(12) Explain Matching in Bipartite Graphs ?